The cost of uncertainty

Quality-value relationships in the signal detection model

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Are more accurate forecasts always more economically valuable?

- No: more accurate forecasts can be less valuable…
  - Depends on measure of “accuracy”
  - Depends on how forecasts are used in decision-making
  - Using the ***optimal decision threshold*** is critical to getting maximum value out of forecasts

Methods from signal detection theory...

- Provide measures of accuracy independent of decision threshold/calibration
- Also imply a model for the dependence on decision threshold of measures of forecast quality and of expected value
- Hence can be used to model dependence of value on decision threshold and accuracy

Decision threshold…

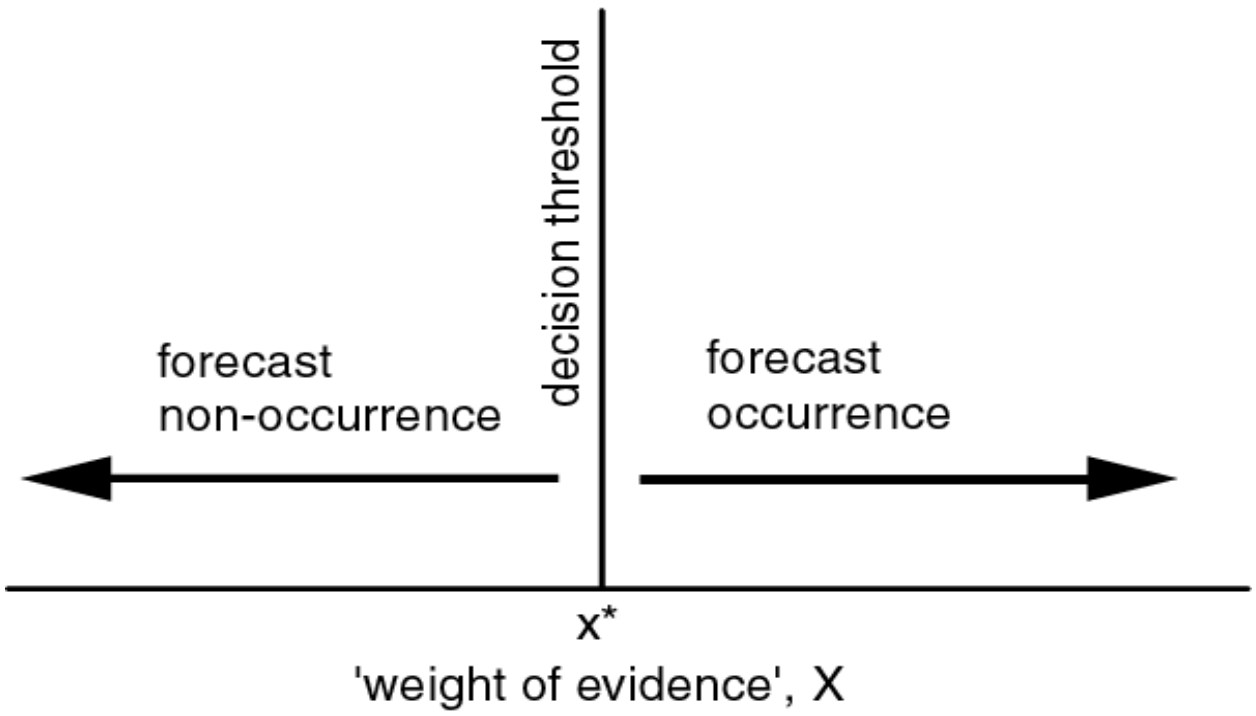
- Level of certainty at which decision changes
- May be stated as a threshold probability for some critical event
- Probability estimated by forecaster on the basis of his judgement of the current weight of evidence for the event

Simple forecast/decision situation

- Yes/no forecasts
- Decisions based only on forecasts
- Static (single-stage) situation

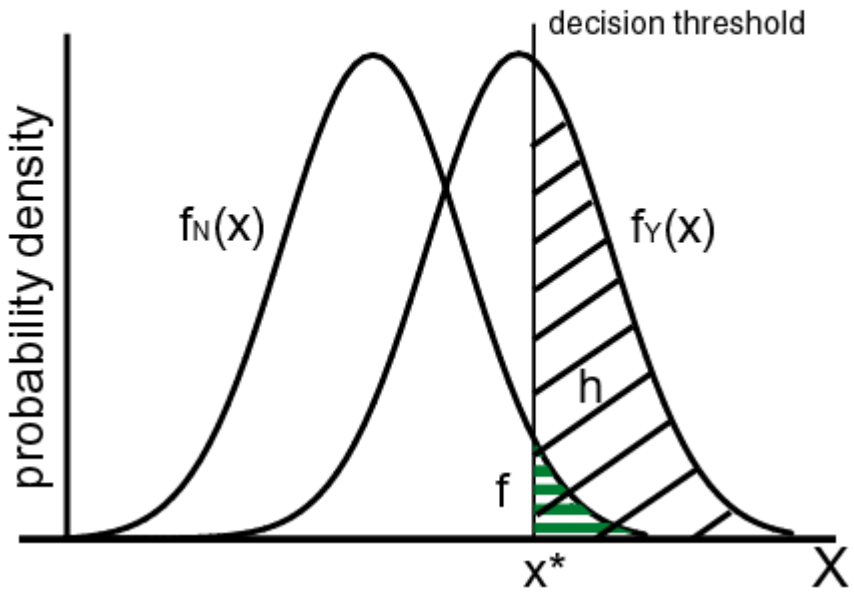
Choosing the forecast

Figure 1

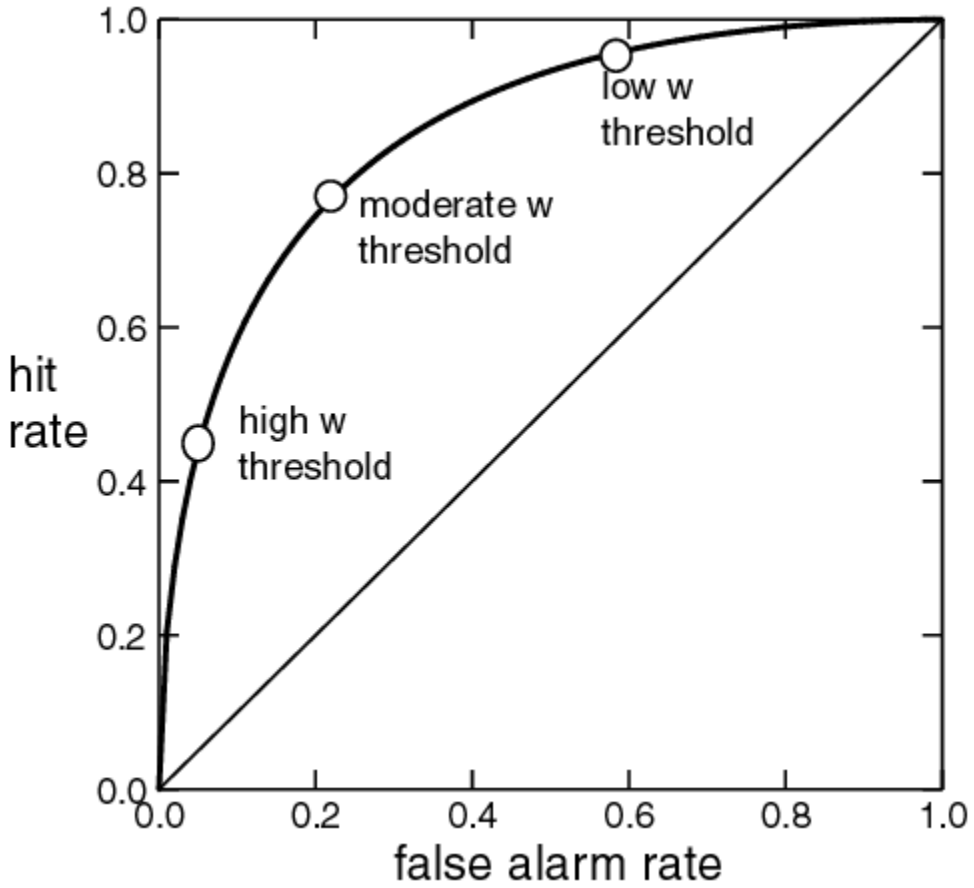


Hit and false alarm rates in the signal detection model

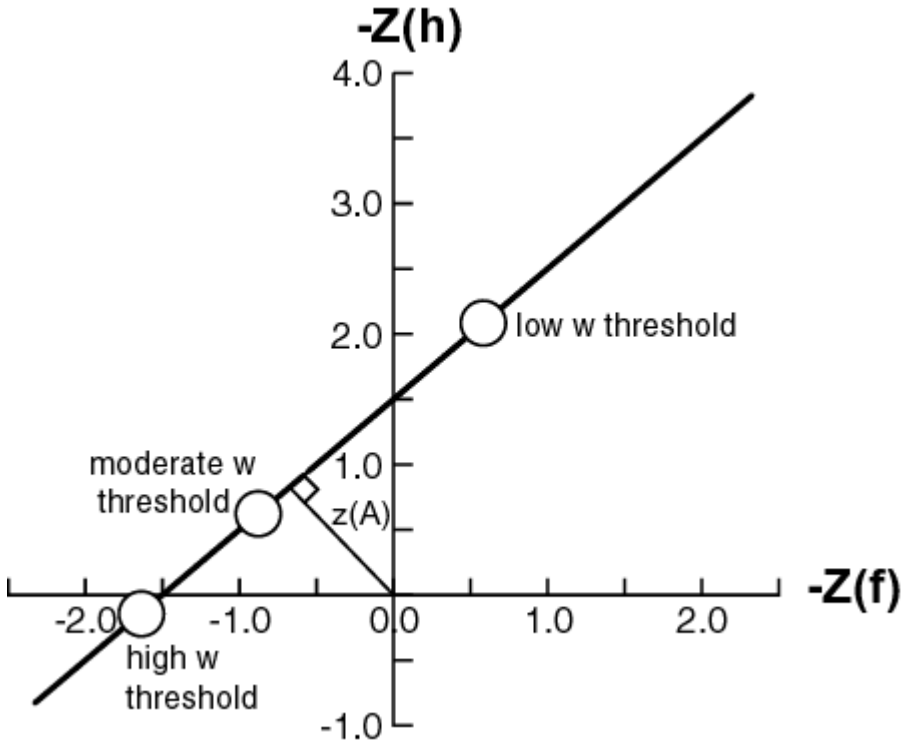
Figure 2



ROC
linear probability axes

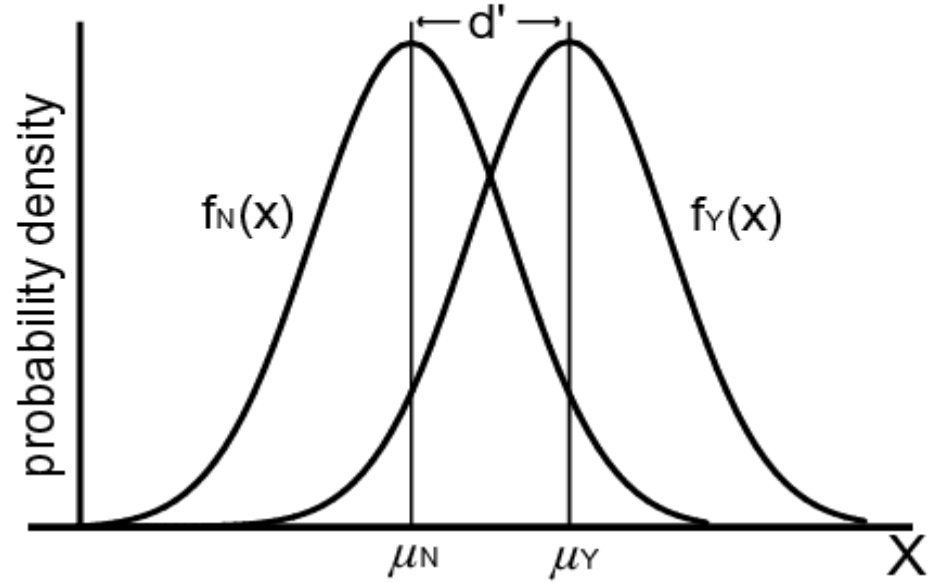


ROC
axes transformed to s.n.d.

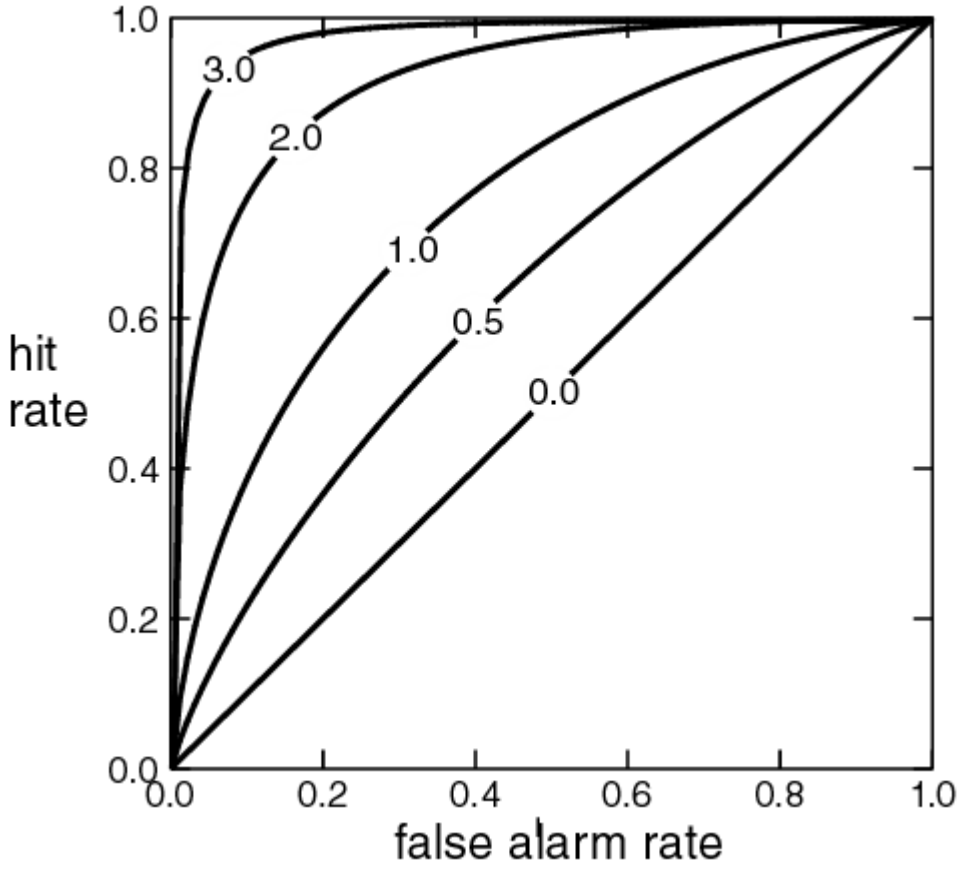


A measure of discrimination for Y/N forecasts

Figure 3



Equal-variance ROCs labels d’

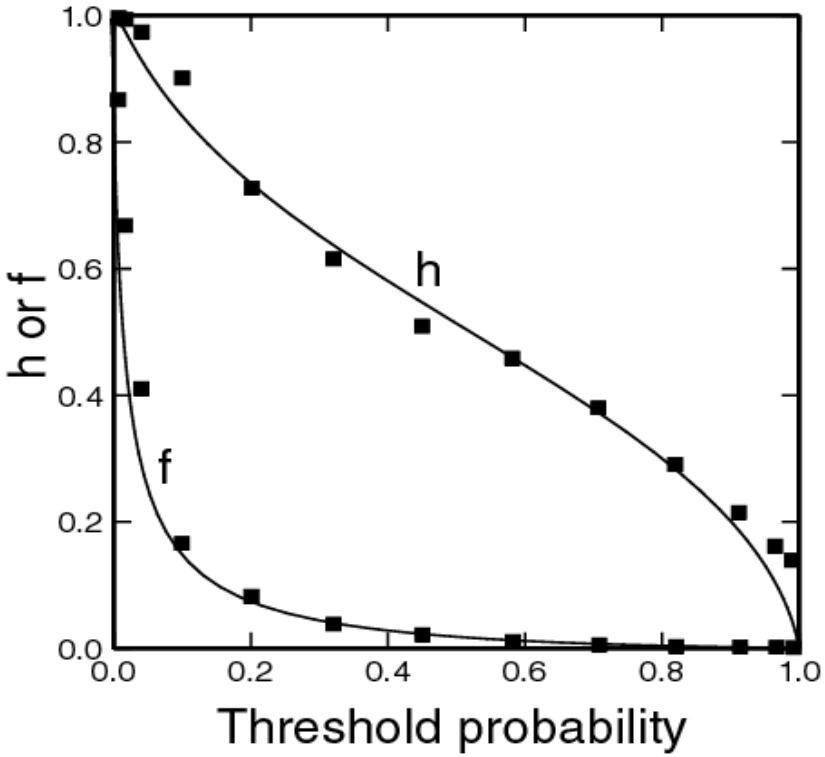


Some definitions

- hit rate, $h = Pr(F=y|E=y) = 1-Pr(F=n|E=y)$
- false alarm rate,
  $f = Pr(F=y|E=n) = 1-Pr(F=n|E=n)$
- sample climate, $p_c = Pr(E=y) = 1-Pr(E=n)$

Empirical relationships h(p*) and f(p*)

Figure 11



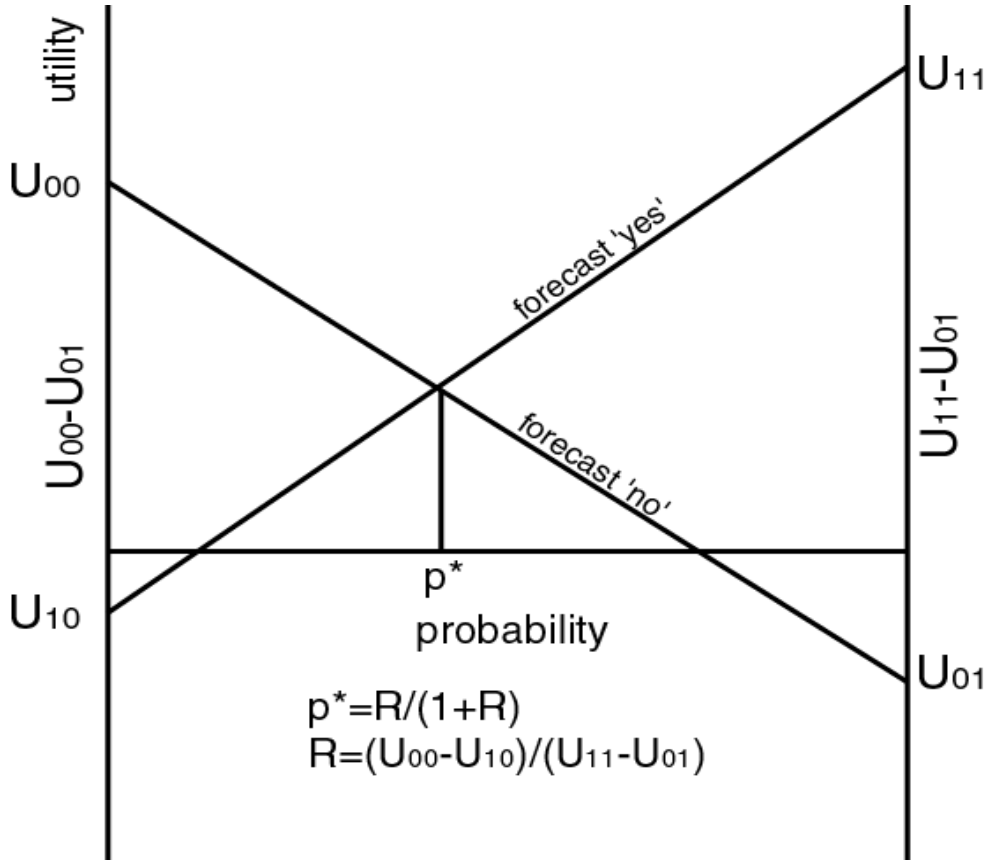
Joint distribution of forecasts and observations

		EVENT	
		NO	YES
FORECAST	NO	True Negative (correct rejection) (TN)	False Negative (miss) (FN)
	YES	False Positive (false alarm) (FP)	True Positive (hit) (TP)

Utility matrix for decisions

		EVENT	
		NO	YES
ACTION	A0	U_{00}	U_{01}
	A1	U_{10}	U_{11}

Optimal threshold probability



Expected value of forecasts

- $EU = Pr(TN)U_{00} + Pr(FN)U_{01} + Pr(FP)U_{10} + Pr(TP)U_{11}$
- $EU = (1- p_c)(1-f)U_{00} + p_c(1-h)U_{01} + (1- p_c)fU_{10} + p_chU_{11}$
- $EU_{perf} = (1- p_c)U_{00} + p_cU_{11}$
- $CoU = EU_{perf} - EU$

  $=(1- p_c)f(U_{00}-U_{10}) + p_c(1-h)(U_{11}-U_{01})$

Relative cost

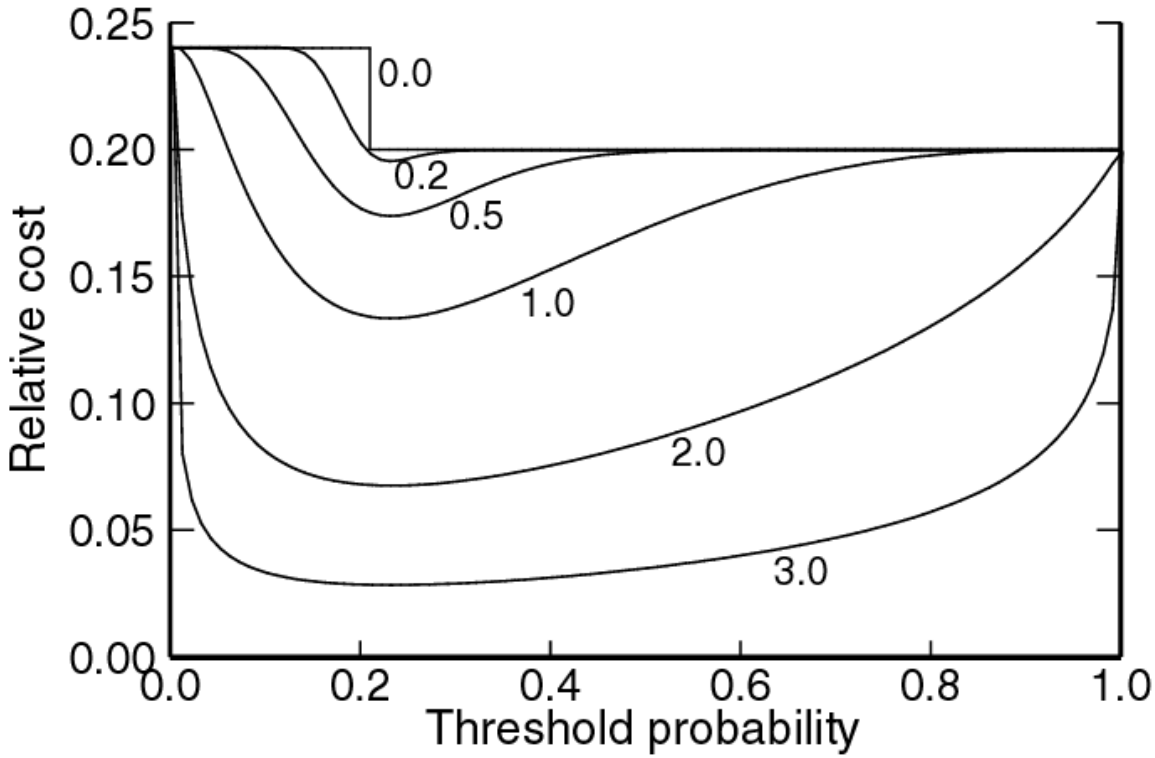
- $C = CoU/(U_{11}-U_{01})$
  $= (1- p_c)fR + p_c(1-h),$
  - $R=(U_{00}-U_{10})/(U_{11}-U_{01})$
- Reduction in value from that of perfect forecasts
- Can be graphed as a function of threshold probability, because the variation of h and f with threshold probability is known

Results...

- Dependence of relative cost on threshold
- Dependence of relative cost on ‘skill’

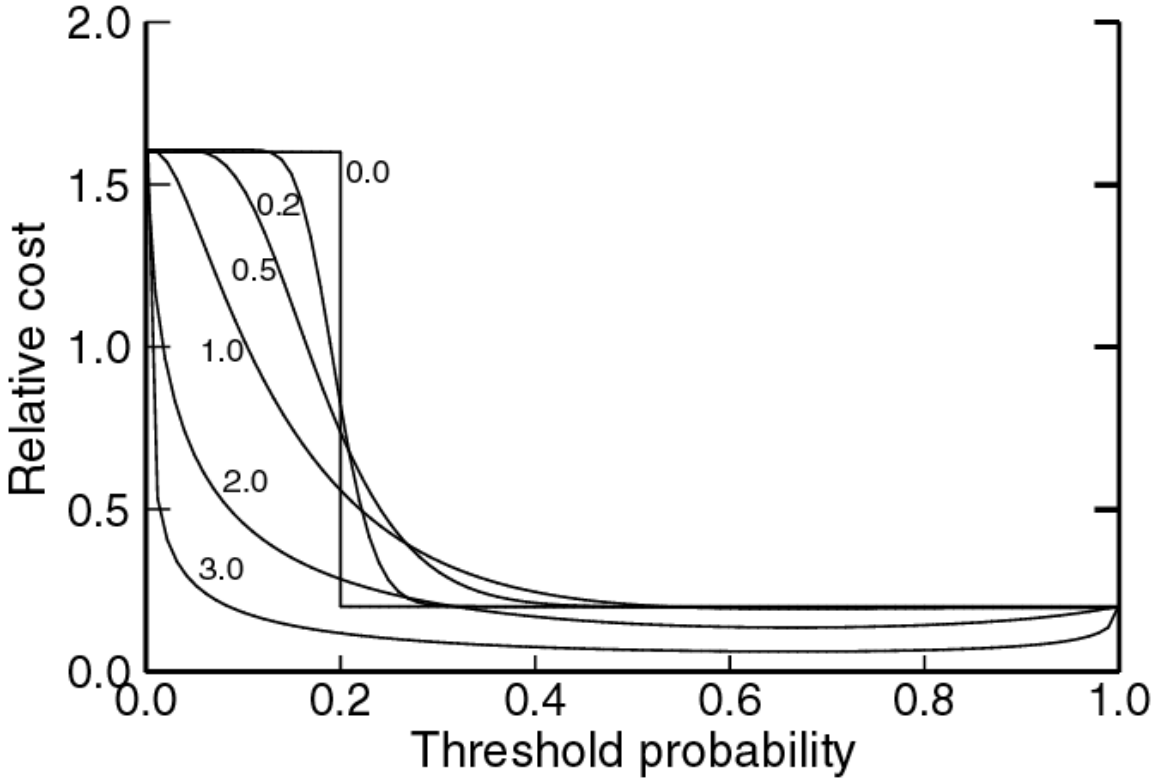
Variation of relative cost with threshold probability for $p^*_{opt} > p_c$, for d' from 0.0 to 3.0. R=0.3, p_c=0.2, p^*_{opt}=0.23.

Figure 4



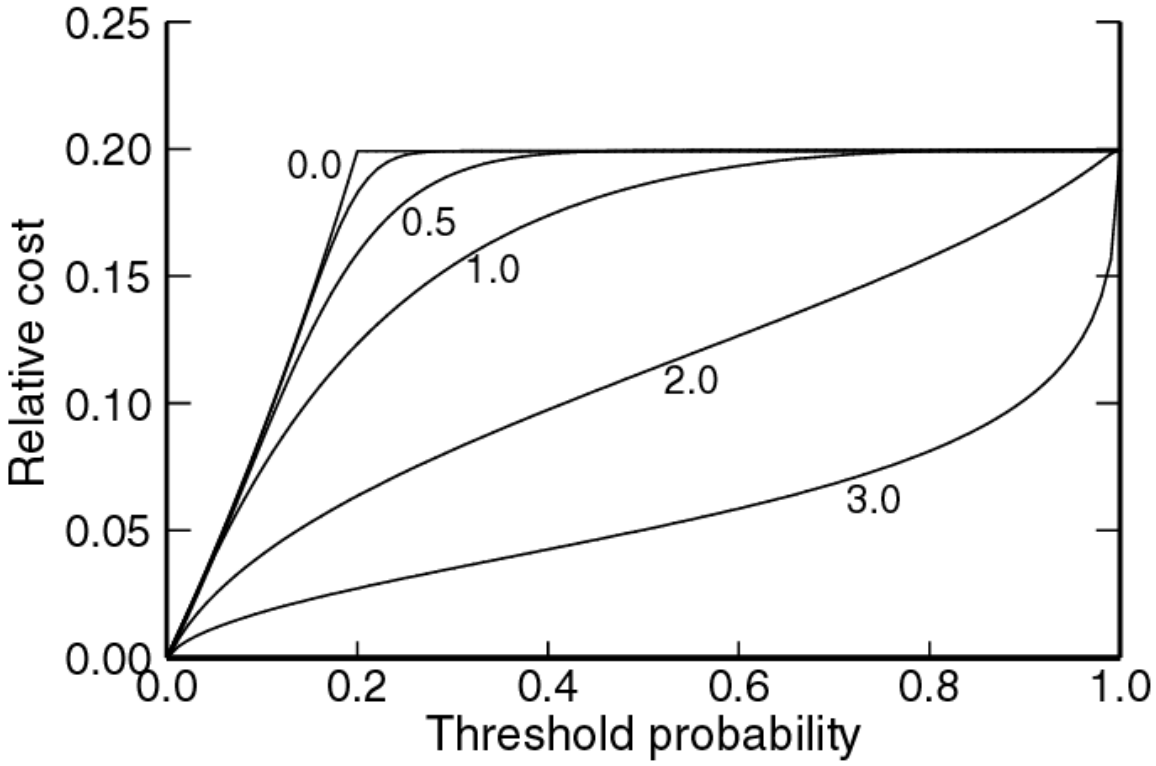
Variation of relative cost with threshold probability for $p^*_{opt} \square p_c$, for d' from 0.0 to 3.0. R=2.0, p_c=0.2, p^*_{opt}=0.67

Figure 5



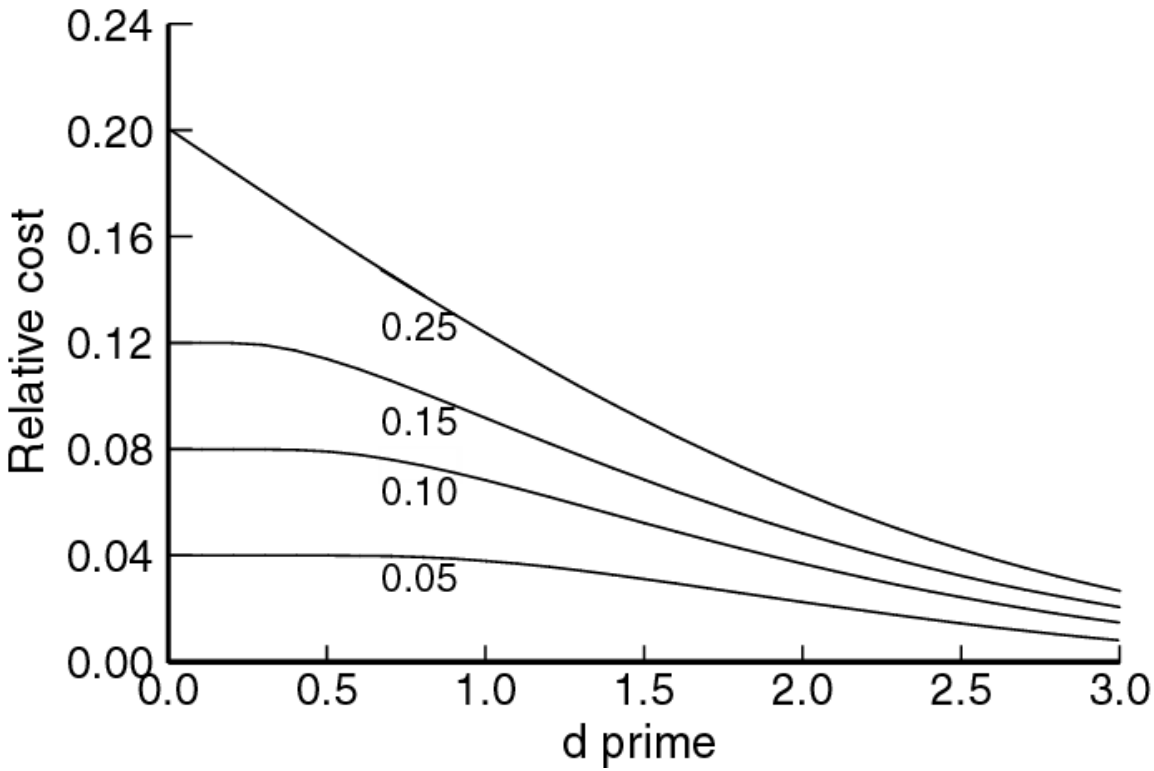
Variation of relative cost with threshold probability for optimal decisions; for d' from 0.0 to 3.0. p_c=0.2

Figure 6



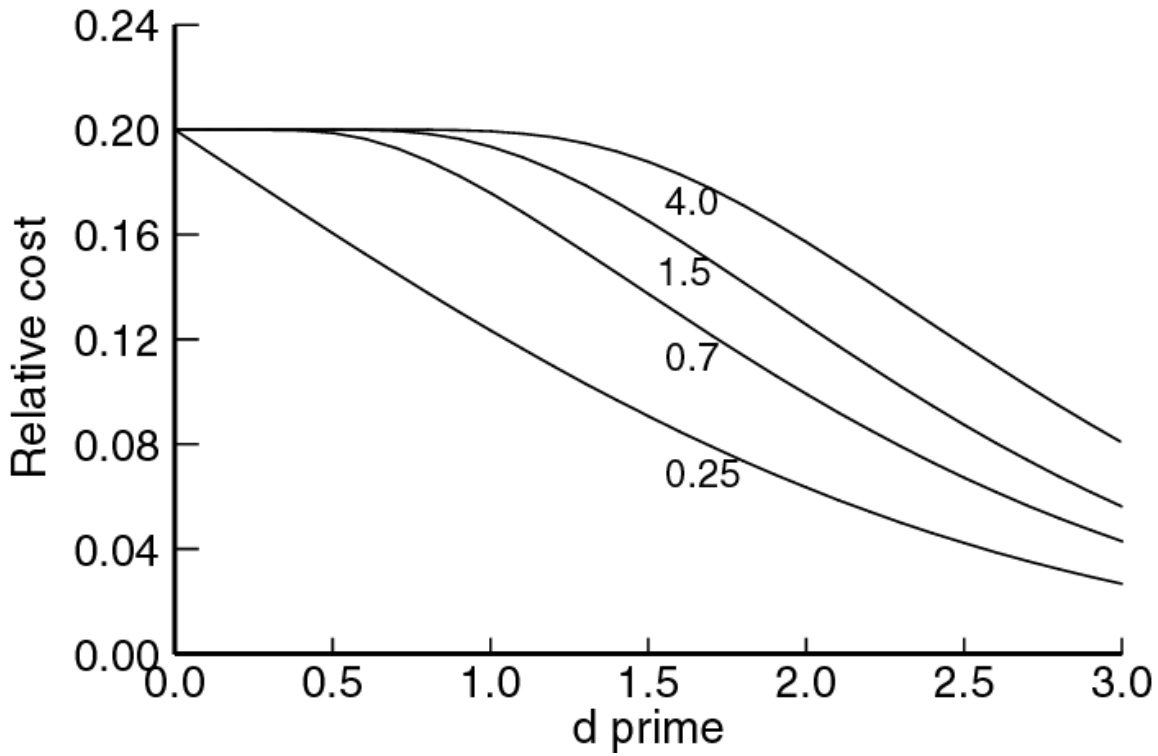
Variation of relative cost with d' for optimal decisions and $p^*_{opt} <= p_c$; for R from 0.05 to 0.25. p_c=0.2.

Figure 7



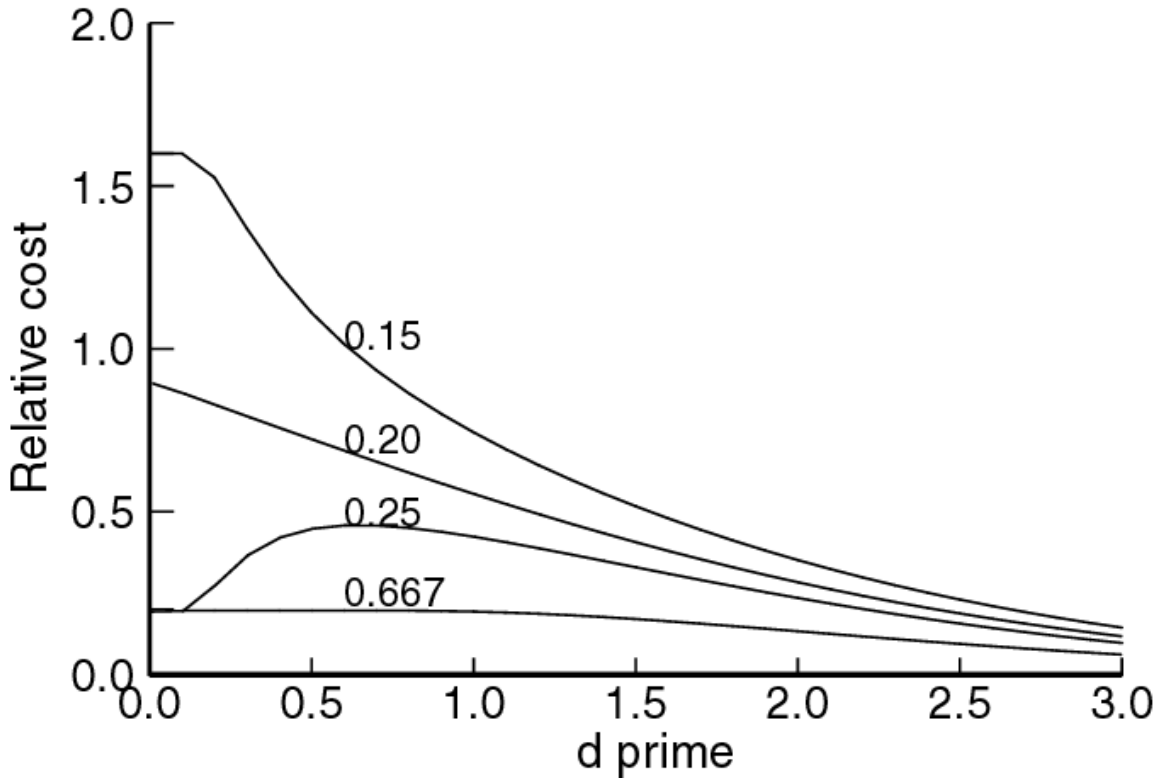
Variation of relative cost with d' for optimal decisions and $p^*_{opt} >= p_c$; for R from 0.25 to 4.0. p_c=0.2

Figure 8



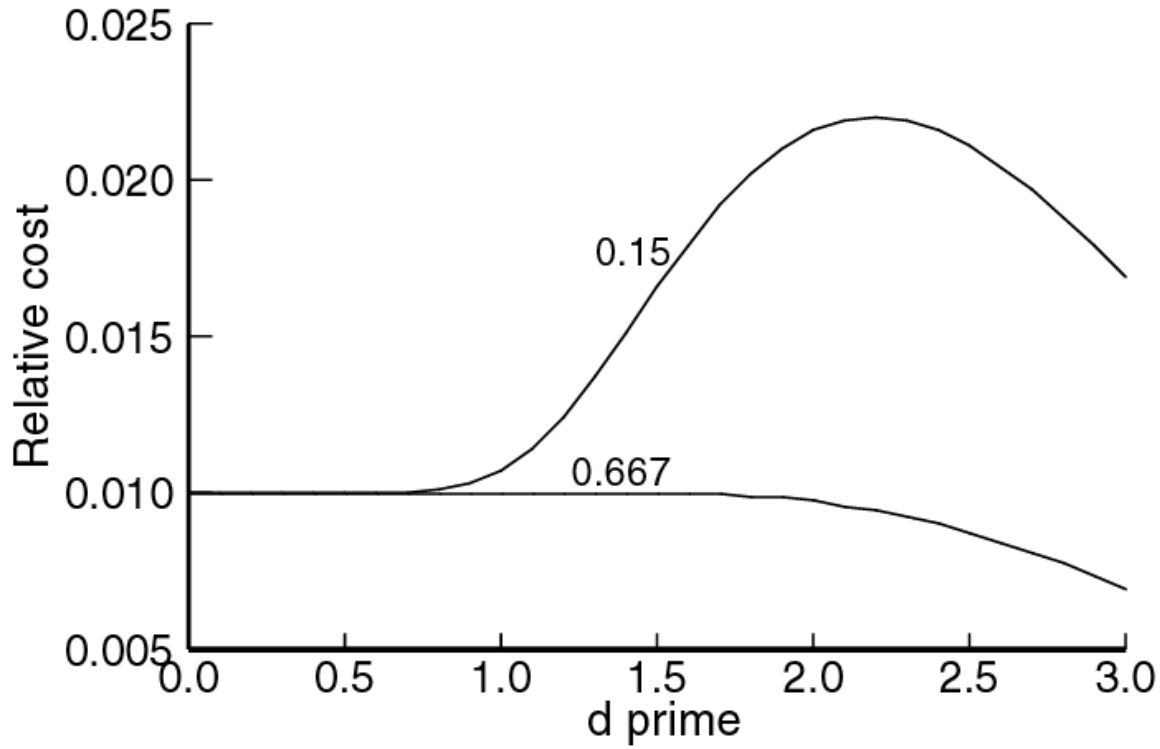
Variation of relative cost with d' for suboptimal decisions; for p* from 0.15 to 0.667 p_c=0.2, R=2.0, p^*_{opt}=0.667

Figure 9

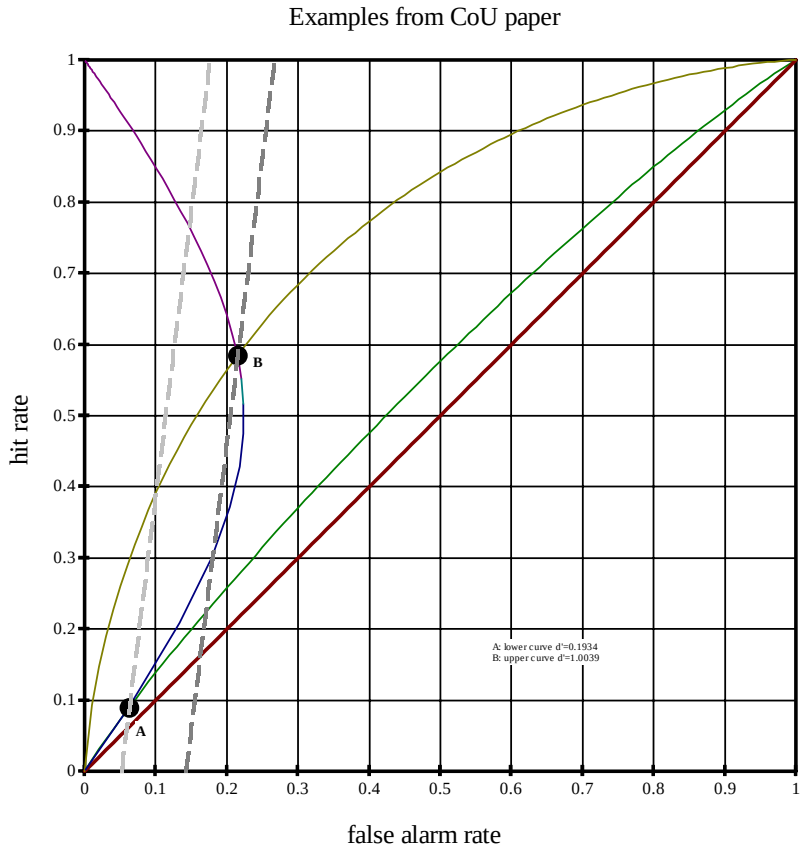


Variation of relative cost with d' for suboptimal decisions and a ***rare event***, for p*=0.15 and p*=0.667. p_c=0.01, R=2.0, p*opt=0.667

Figure 10



Expected costs on ROC examples



Summary...

- The value of forecasts depends on
  - intrinsic skill of forecasts
  - decision threshold for action
- Decision threshold is as important as skill
  - there is an optimal threshold which depends on the structure of the decision situation Uij
  - poor choice of decision threshold can wipe out economic benefits of skill

...Summary

- The relationship between hit and false alarm rates and decision threshold provides a useful approach to modelling the economic value of forecasts